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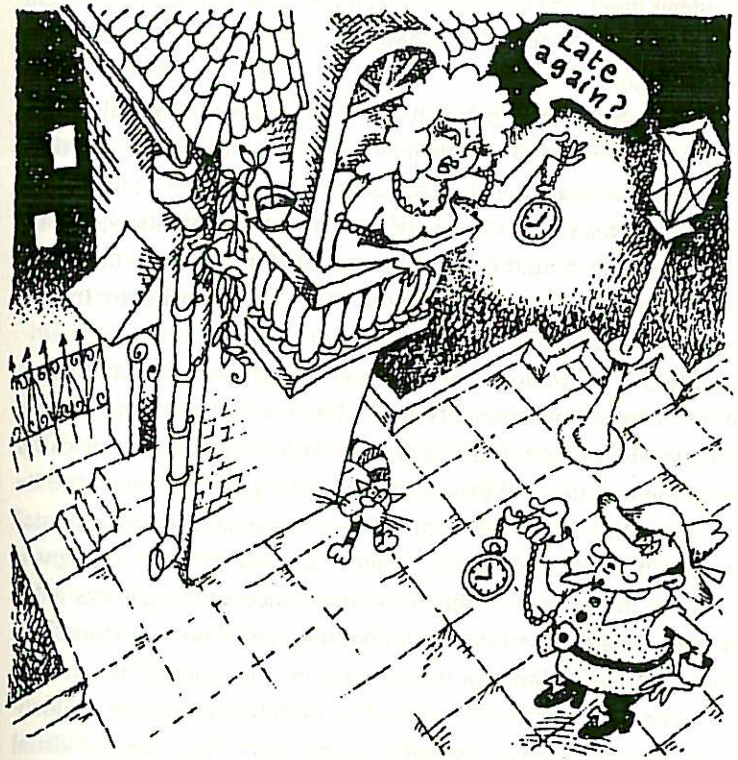
Time, space and gravitation



Everyone knows that the space of the Universe is three-dimensional. This means that space is characterized by length, width and height. The same is true for any body. Somewhat differently, the position of a point in space is characterized by three numbers known as coordinates. If we draw straight lines or planes or complicated curves through space, their properties are described by the laws of geometry. These laws have been known to man since ancient times and were compiled by Euclid in the 3rd century BC. Euclidean geometry is studied in schools as a harmonious system of axioms and theorems that describe all properties of lines, surfaces and solids.

If we wish to study not only the spatial position but also processes occurring in three-dimensional space, we need to add time as well. An event taking place at some point is characterized by the position of this point, that is, by indicating three numbers, and by a fourth number, that is, the moment of time at which the event occurred. For the event the time is its fourth coordinate. In this sense we say that our world is four-dimensional.

All this is well known, of course. Then why wasn't this formulation of four-dimensionality treated as serious and fraught with new knowledge before the theory of relativity was born? The catch lay in the fact that the properties of space and time seemed to be too dissimilar. When we speak of space, we have a static mental picture in which bodies or geometric figures are fixed at a given moment. In contrast to this, time flows incessantly (always from the past towards the future) and bodies change their positions.

Space is three-dimensional but time is one-dimensional. In fact, time was compared to a straight line even by the ancient philosophers, but this always seemed to be no more than a useful visual image without any profound meaning. Things changed drastically after relativity theory was discovered.

In 1908, the German mathematician Hermann Minkowski, developing further the ideas of this theory, said: 'From now on, space as such and time as such must turn into fictions and only some form of combining them together will retain independence.' What did Minkowski mean in this forthright and categorical declaration?

He wished to emphasize two aspects. Firstly, that time intervals and spatial lengths are relative, depending on the choice of the reference frame. Secondly – and this was the more important part of his words – that space and time are connected inseparably. In fact, they are two facets of a unified entity: four-dimensional spacetime. The pre-Einstein physics knew nothing of these close ties. What are their manifestations?

The most important one is that spatial intervals can be determined by measuring the time required for light or for any electromagnetic waves to travel the distance we wish to know. This is the method used in now familiar radar. The essential point is that the velocity of propagation of any electromagnetic waves is completely independent of the motion of the source or that of the body reflecting the waves, and always equals c. Hence, the distance is found simply by multiplying the constant velocity c by the time of travel of the electromagnetic signal. It was not known before the arrival of Einstein's theory that the velocity of light is constant and thus it was expected that this procedure would be wrong.

Of course, one can choose the opposite approach, that is, measure time by a light signal covering a known distance. For example, if we make a light signal shuttle between two mirrors spaced by three meters, each jump would last one one-hundred-millionth of a second. The number of times that this unusual light pendulum has swung between the two mirrors is the number of one-hundred-millionths of a second that has elapsed.

These examples illustrate the relationship between time and space. Their respective intervals differ only in a constant, familiar multiplier 'c'.

Another, at least as important manifestation of the unity of space and time is that as the velocity of a body increases with time, the rate of advance of time decreases for the body in exact correspondence with the reduction of its longitudinal dimension (along the direction of motion). Because of this exact correspondence of these two quantities – the distance in space between two events (e.g. flashes of two light bulbs) and the time interval separating them, a simple calculation yields the quantity that is constant for all observers, regardless of the velocity at which they move, and that is independent of the velocity of any two 'laboratories'. This quantity plays the role of distance in four-dimensional spacetime. The spacetime is precisely the 'unification' of space and time announced by Minkowski.

It may not be too hard to comprehend this formal unification of space and time. Imagining the four-dimensional world is far more difficult. The difficulty is not surprising. When we draw geometric figures in a plane, we usually encounter no difficulties in projecting what we want; these figures are two-dimensional (only have a length and a width).

Quite a few people have a hard time imagining three-dimensional forms in space – pyramids, cones, planes intersecting them etc. As for creating an image of four-dimensional forms, it is a very demanding task even for experts who work with relativity theory all the time.

I will quote the very famous British physics theoretician Stephen Hawking, an expert of incomparable standing in relativity theory. He says in his famous book *A Brief History of Time*: 'I personally find it hard enough to visualize three-dimensional space!' Which shows that the reader defeated by imagining four-dimensional world need not be unhappy. Experts use the spacetime concept quite successfully. For instance, the motion of a body can be shown by a line in spacetime.

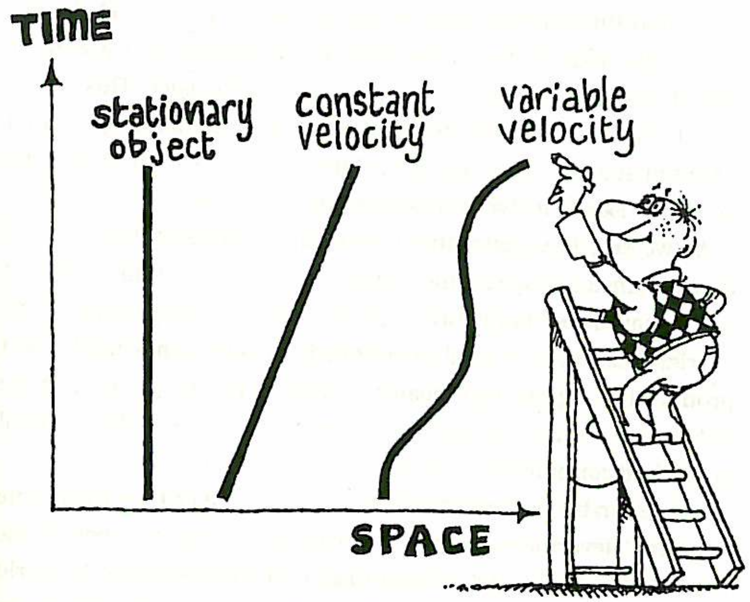


Fig. 6.1.

In figure 6.1 the distance in space in one direction is plotted along the horizontal axis and time is plotted along the vertical axis. We can mark the position of a body at each moment of time. If the body is at rest in our 'laboratory', that is, if its position does not change, our plot shows this by a vertical line. If the body moves at a constant velocity, we obtain a tilted straight line. An arbitrary motion produces a curved line, known as the *world line*. In the general case, one has to imagine that a body can move in the other two directions, not only along one axis. Its world line will picture the existence of the body in four-dimensional space.

This figure is an attempt to show that space and time enjoy identical status. The values they assume are merely marked on different axes. Nevertheless, there exists an essential difference between space and time: we can stop in space, but not in time. The world line of the body is drawn vertically in our figure: as if a body is pulled along by time flow, even if it is at rest in space. This is true for all objects in the Universe: their world lines cannot stop, cannot be cut at some moment of time, since time never freezes. As long as a body exists, its world line stretches on.

As we see, there is nothing mystical in physicists' concept of four-dimensional spacetime. Albert Einstein once remarked that a non-mathematician is often given to mystical trepidation when hearing 'four-dimensional' mentioned – a feeling not unlike that produced by a ghost in a theater... while in fact no phrase is more banal than saying that the world around us is a four-dimensional spacetime continuity.

There can be no doubt that a new concept takes time to become habitual. Nevertheless, physics theorists use the concept of the four-dimensional world as their daily tool, manipulating the world lines of bodies, calculating their lengths, finding their intersection points etc. In this four-dimensional world, they develop four-dimensional geometry which is similar to Euclidean geometry. To honor Hermann Minkowski, the four-dimensional world is known as Minkowski spacetime.

Having created relativity theory in 1905, Albert Einstein worked very hard for ten more years trying to connect his theory with Newton's universal law of gravitation.

The law of gravitation as formulated by Isaac Newton is incompatible with relativity. Indeed, Newton's statement declares that the force with which a body attracts another body is inversely proportional to the squared distance between them. Therefore, if the attracting body is displaced, the distance between the bodies changes and this will alter instantaneously the attractive force acting on the other body. Therefore, Newton's gravitational force propagates through space at infinite speed. Relativity states, however,

that this simply cannot be. The speed at which any force, any effect can be transferred cannot exceed the velocity of light, so that gravitation cannot act instantaneously!

In 1915 Einstein completed the development of a new theory which joined together relativity and gravitation. He called it general relativity. Subsequently, the theory developed in 1905, which could not deal with gravitation, was referred to as special relativity.

The mathematical tools of the new theory proved very complex and unorthodox for physicists of the time; as a result, it was not immediately understood nor accepted by a large number of theorists.

Despite the complexity of the mathematics involved, the basic ideas are simple (as everything that is truly important), although they were extraordinarily brave and changed the concept of space and time in an even more drastic way than special relativity did.

Isaac Newton perfectly understood that he was only able to describe the law of gravitation but that he failed to comprehend specifically how gravitation propagates from body to body, what was, so to speak, the 'mechanism' of functioning of gravitation. Newton wrote: 'But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses (*Hypotheses non fingo*); for whatever is not deduced from the phenomena is to be called an hypothesis.' It was sufficient for him at that stage that gravitation does exist and acts in accordance with the laws as he formulated them, and that they provide adequate explanation for all motions both of heavenly bodies and of the sea.

Einstein's general relativity does reveal the 'mechanism' of gravitation. It states that gravitation is dramatically different from all other forces of nature. To clarify this point, let us resort to the following analogy. A sphere rolling on a flat surface moves along a straight line, which is the shortest line connecting any two points.

If the sphere is made to roll on a curved surface, it has to follow a curvilinear trajectory, because it is impossible to place a straight line on a curved surface. For instance, if a ball is rolling on the surface of the Earth (we assume its surface to be absolutely smooth, without mountains, valleys or obstacles), it follows the shortest trajectory on the sphere (such lines drawn on any curved surface are known as *geodesics*).

Einstein's theory of gravitation states that gravitating bodies geometrically distort the spacetime around them. I have already mentioned the difficulties in imagining four-dimensional spacetime, but if it is also curved. . . . However, mathematicians and physicists can live quite well without visualizable concepts. For them, curvature of spacetime constitutes a change in the geometric properties of figures and solids. For example, the ratio of the circumference of a circle to its diameter on a plane is π but this is not so on a curved surface or in a 'curved' space. The geometrical relations in them differ from Euclid's geometry. An expert can operate in such extraordinary space once he knows the laws of the 'curved' geometry.

The discovery that three-dimensional space may be curved was made theoretically at the beginning of the 19th century by the Russian mathematician Nikolai Lobachevsky and at the same time by the Hungarian mathematician Janos Bolyai. At mid-century, a German mathematician working in geometry, Georg Riemann, introduced into mathematics 'curved' spaces with four and even any number of dimensions. From that time on, the geometry of curved space has been known as *non-Euclidean geometry*. The discoverers of non-Euclidean geometries did not know under what specific conditions their geometries might manifest themselves, although some guesses were suggested. The mathematics apparatus that they and their followers developed was later used to formulate general relativity.

Einstein's fundamental idea, therefore, was that gravitating

masses curve the surrounding spacetime. Let us now consider other bodies with very small masses (physicists refer to them as 'probes') which move in this curved spacetime. As before, they move along geodesics. In the non-curved spacetime geodesics are straight lines, but in a curved spacetime they are curvilinear. It is this motion along curved trajectories that we interpret as the motion caused by gravitational forces. The explanation of the gravitational field is thus the 'curved' geometry of spacetime.

Eminent American physicists Charles Misner, Kip Thorne and John Wheeler chose to begin their massive monograph (*Gravitation* 1973 (San Francisco: Freeman), 1279 large-size pages) with the following amusing story.

> Once upon a time a student lay in a garden under an apple tree reflecting on the difference between Einstein's and Newton's views about gravity. He was startled by the fall of an apple nearby. As he looked at the apple, he noticed ants beginning to run along its surface. His curiosity aroused, he thought to investigate the principles of navigation followed by an ant...
>
> His eyes fell on two ants starting off from a common point *P* in slightly different directions. Their routes happened to carry them through the region of the dimple at the top of the apple, one on each side of it. Each ant conscientiously pursued his geodesic. Each went as straight on his strip of appleskin as he possibly could. Yet because of the curvature of the dimple itself, the two tracks not only crossed but emerged in very different directions.
>
> "What happier illustration of Einstein's geometric theory of gravity could one possibly ask?" murmured the student. "The ants move as if they were attracted by the apple stem... Now I understand better what this book means."

The authors concluded:

> Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve.

Everything is extremely unusual in this story: a curved four-dimensional spacetime that cannot be visualized, the interpretation of the force of gravitation in terms of geometric factors. For the first time, physics is directly linked to geometry. Looking closely at physics' successes, we notice that as we come closer to our time, its discoveries become less and less conventional while its notions become less and less amenable to visualization. Well, there is nothing to be done about it: nature is extremely complex and we have to accept that the deeper our penetration into the realm of its secrets, greater and greater is the effort required for the process, including the efforts of our imagination. The word 'accept' may not be the right one here; one would rather like to emphasize that the going is getting more and more exciting even if harder and harder.

The reader will profit from information on two other facts from Einstein's gravitation theory.

In Newton's theory, the field of gravitation is determined exclusively by the mass of the body creating the field. According to Einstein's theory, all types of energy take part in creating gravitation, including energy connected with pressure and tension of the body, and the electromagnetic field. The second important prediction of the theory is that if the gravitating masses move with an acceleration, they must emit gravitational waves: we know that accelerated electric charges emit electromagnetic waves. (It is rather unfortunate that I have no chance of going into details of what gravitational waves are.)

Both these predictions of Einstein's theory, which immediately distinguish it from Newton's theory, manifest themselves only under very exotic conditions, while under ordinary conditions the effects stemming from these predictions are extremely weak and utterly undetectable. In a conventional environment Einstein's theory is practically indistinguishable from Newton's theory. On the other hand, Einstein's theory leads to conclusions completely different from anything implied by Newton's theory in very strong gravitational fields or in fields that rapidly vary in time. This will be the subject of later discussion.

Immediately after formulating his theory, Einstein pointed out three effects which, although very minute under usual circumstances, can nevertheless be put to the test in astronomical observations and used to confirm or disprove the new theory.

The first two effects involve small deviations from the calculations of Newtonian physics in the motion of planets orbiting the Sun and in the trajectories of light passing very close to it. A comparison with the observational data did reveal these effects and completely confirmed the conclusions of the new theory. By the way, the observation of Einstein's effects demonstrated that the space in the vicinity of the Sun is indeed slightly curved and its geometry somewhat deviates from Euclidean geometry.

The third effect deals with time and therefore I will go into more details here.

Einstein's theory predicts that time flows more slowly in a strong gravitational field than outside it. This means, for example, that on the surface of the Sun any clock runs more slowly than on the Earth, since gravitational pull is much stronger on the Sun. For the same reason, a clock lifted high above the surface of the Earth ticks slightly faster than a clock on the surface itself.

A considerable number of experiments were conducted to detect and quantify this exciting effect; I will describe some of them. Let us start with the observations of slowdown on the Sun.

The objects that served as 'clocks' were atoms of chemical elements. Absorption lines in the solar spectrum due to these atoms correspond to certain frequencies of oscillation of electrons, when these electrons jump from one atomic energy level to another. If time on the Sun does flow at a slower pace, the frequencies of these oscillations must decrease and therefore, the lines in the spectrum must shift towards the red end. The shift is extremely small, since the relative slowdown of time on the Sun is by only one part per two million. Hence, the frequency of a spectral line should shift towards the red end of the spectrum by the same fraction. This effect is known as the *gravitational red shift*. The experiment was to measure just this tiny shift. Astronomers would be able to measure the gravitational red shift reliably were it not for the complicating effects caused by the motion of masses of gas on the solar surface.

Unfortunately, turbulent motion of solar gas masks the gravitational effect owing to the Doppler effect, so that astronomers faced serious difficulties. The first attempts, made immediately after the prediction was formulated, were rather unsuccessful; only relatively recently, during recent decades, has analysis of the solar spectrum yielded complete confirmation of the theory. Even though the difference between the rates of time flow on the Earth and the Sun is negligibly small, the difference between the number of years that have elapsed on these two bodies is quite considerable. Both are known to have existed for about five billion years, but the Earth has clocked ten thousand years more than the Sun...

In 1968 the American physicist Irvin Shapiro measured the retardation of time flow on the surface of the Sun by a very ingenious method. He was conducting radar measurements of Mercury when this planet was on the part of its orbit around the Sun which is diametrically opposite the Earth. The radar beam going towards Mercury and the reflected signal had to pass close to the Sun, and thus took slightly longer to cover the distance than it did when Mercury was not hiding behind the Sun. This time delay (about one ten thousandth of a second) was indeed reliably measured.

Astronomers know stars which are much denser than the Sun, so that the gravity field at their surface is very much stronger: these are neutron stars and white dwarfs. Observations of the time retardation effect for the light emitted by them also confirmed the theory. Note that on the surface of neutron stars, time flows twice as slowly as on the Sun!

TIME, SPACE AND GRAVITATION

It is especially impressive that the slowdown in the flow of time in the gravitational field has been measured on the Earth, in laboratory conditions. This was achieved in 1960 by the American physicists Robert Pound and Glen Rebka. They compared the rate of time at the base of a tower and at a height of 22.6 m, where the clock was expected to run slightly faster. The 'clock' was in fact a set of extremely accurate instruments using the phenomenon of emission of gamma rays of precisely known frequency under certain conditions. The theory predicted a fantastically small difference between the clock rates at two heights: three ten-thousandths of one billionth of a second. Nevertheless, the difference was measured and confirmed the theory!

Sixteen years later, similar experiments were repeated but under very different conditions. In one of them, an instrument emitting radiation at a prescribed frequency (known as the hydrogen frequency standard) was launched by a rocket to a height of about ten thousand kilometers. At such an altitude, time runs faster than on the Earth's surface again by the minutest amount but the difference between rates is nevertheless one hundred thousand times greater than in the Pound and Rebka experiment. The experiment (the rocket flight) lasted two hours. However, the flight was preceded by five years of intensive work. Einstein's formula was shown to hold to within two hundredths of one percent!

At about the same time, direct experiments were carried out with clocks, or rather with super-accurate atomic clocks.

Italian physicists moved several such 'clocks' on a truck high into the mountains, and several hours later brought them back, to compare their readings with the clock that stayed below all the time. This stationary reference clock was found to lag behind, in complete accordance with Einstein's theory (the difference was measured in nanoseconds, that is, in billionths of a second).

In a similar American experiment an atomic clock was placed in an airplane which was kept in flight at an altitude of nine kilometers for fourteen hours. After landing, the readings of the clock were compared with the reference clock on the Earth's suface. Einstein's theory was again beautifully confirmed.

There is thus no doubt that time is slowed down in the gravitational field. In most of these cases, the changes are almost immeasurably small, but we will see that astronomers and physicists know situations in which the difference between time rates is colossal.

General relativity has completely reshaped our ideas of space and time. Neither is an invariable scene on which the dramatic history of the Universe is acted out. Space is not an infinite rigid skeleton. Moving matter is constantly warping it, changing its geometric properties. The acceptable part of the naive notions of our predecessors about the all-encompassing, immutable time river is gradually dwindling. As we see now, it does not flow everywhere with equal grandeur: it is rapid in gorges but slow over shallows; we will see later how it splits into numerous streams, brooks and rivulets, which move forward at different speeds, depending on 'local' conditions.